Bachelor of Science (B.Sc.) Semester–VI (C.B.S.) Examination
MATHEMATICS
(M-12 : Discrete Mathematics and Elementary Number Theory)
Optional Paper—2

Time—Three Hours] [Maximum Marks—60

N.B. :— (1) Solve all the **FIVE** questions.

(2) All questions carry equal marks.

(3) Question Nos. **1** to **4** have an alternative. Solve each question in full or its alternative in full.

UNIT—I

1. (A) Prove that in a lattice, if $a \le b \le c$, then :

(i) $a \oplus b = b * c$

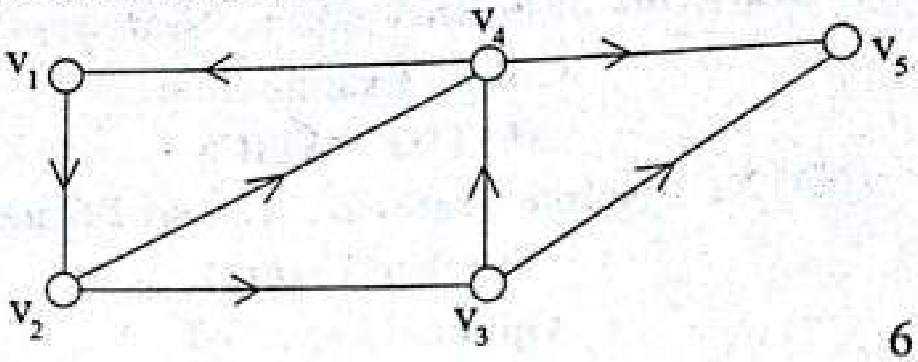
(ii) $(a * b) \oplus (b * c) = b$

(iii) $(a \oplus b) * (a \oplus c) = b$. 6

(B) Find complements of every element of the lattice (S_n, D) for $n = 75$. 6

OR

1. (C) Find all the in degrees and the out degrees of the digraph given below. Also find all the elementary cycles of this graph :



6

(D) Let R be symmetric and transitive relation on the set A. Show that if $\forall\ a \in A$, there exists $b \in A$ such that $(a, b) \in R$, then R is an equivalence relation. 6

UNIT—II

2. (A) Prove that give integers a, b with $a > 0$, there exists unique integers q and r such that $b = aq + r$ with $0 \le r < a$. 6

(B) Find the greatest common divisor and the least common multiple of 482 and 1687. 6

OR

2. (C) Prove that if $a \equiv b \pmod{m}$ and $c \equiv d \pmod{m}$, then $ax + cy \equiv bx + dy \pmod{m}$. 6

(D) If $a \equiv b \pmod{m}$, then using mathematical induction, prove that $a_n \equiv b_n \pmod{m}$ for positive integer n. 6

UNIT—III

3. (A) Let p be an odd positive integer and 'a' be an integer with (a, p) = 1. Then prove that $\left(\frac{a}{p}\right) \equiv a^{\frac{p-1}{2}} \pmod p$. 6

(B) If p and q are odd primes and one of which is of the form 4k +1, then prove that $\left(\frac{p}{q}\right) = \left(\frac{q}{p}\right)$. 6

OR

3. (C) Solve the congruence, if it is solvable :

$x^2 \equiv 7$ (mode 31). 6

(D) Find all primes p such that :

$\left(\frac{5}{p}\right) = -1$. 6

UNIT—IV

4. (A) Find all the solutions in positive integers of the equation 5x + 3y = 52. 6

(B) Find all the primitive Pythagorean triples x, y, z such that z – y = 1. 6

OR

4. (C) Find all the primitive solutions of $x^2 + y^2 = z^2$ with the condition $0 < z < 30$. 6

(D) Prove that every terms in a Farey sequence is in reduced form. 6

Question—V

5. (A) If $R = \{(1, 2), (2,3), (1,3)\}$, then find domain and range of R. 1½

(B) Prove that $(1 * a) \oplus (0 * a') = a$. 1½

(C) Prove that $a|b$ and $b|c \Rightarrow a|c$. 1½

(D) Show that $20x \equiv 4 \pmod{30}$ is not solvable. 1½

(E) Find quadratic residues of 9. 1½

(F) Define Legendre's symbol. 1½

(G) Prove that terms in a Farey sequence are in monotonically increasing order. 1½

(H) Define primitive Pythagorean triplet with an example. 1½

Bachelor of Science (B.Sc.) Semester–VI (C.B.S.) Examination
MATHEMATICS
(M-12 : Differential Geometry)
Optional Paper—2

Time—Three Hours] [Maximum Marks—60

N.B. :— (1) Solve all the **FIVE** questions.

(2) All questions carry equal marks.

(3) Question Nos. **1** to **4** have an alternative. Solve each question in full or its alternative in full.

UNIT—I

1. (A) Find the equation of the normal plane and the tangent line for the twisted cubic $x = at$, $y = bt^2$, $z = ct^2$ at the point $t = 1$. 6

(B) Prove that Darboux vector $\bar{d}$ is constant if K and τ are constant and the $\bar{d}$ has a fixed direction if $\dfrac{K}{\tau}$ is constant. 6

OR

1. (C) Prove that Helices are the only twisted curves whose Darboux vector has a constant direction. 6

(D) Find K and τ at any point θ on the helix :

$x = a \cos \theta$, $y = a \sin \theta$, $z = a\,\theta$. 6

UNIT—II

2. (A) Show that the curvature K_1 of the involute of a curve Γ is given by $K_1 = \frac{K^2 + \tau^2}{K^2(c-s)^2}$, where K and τ are the curvature and the torsion of the curve Γ. 6

(B) Find the involutes and evolutes of the twisted cubic given by $x = u$, $y = u^2$, $z = u^3$. 6

OR

2. (C) Find the envelopes of the family of cones :

$(ax + x + y + z - 1)(ay + z) = ax(x + y + z - 1)$,

where a is the parameter. 6

(D) Explain the meaning of the terms :

Skew ruled surface, developable surface edge of regression. Examine whether the lines given by $x = 2t^2z + 2t(1 - 3t^4)$, $y = -2tz + t^3(3 + 4t^2)$ generate, when t varies, developable or a skew surface. 6

UNIT—III

3\. (A) Determine the unit normals and the fundamental forms of the surface :

$$\bar{r} = (a \cos u, a \sin u, bv).$$ 6

(B) Define direction coefficients on a surface and obtain formulae for the sine and cosine of the angle between two given directions. 6

OR

3\. (C) Obtain Gauss's formulae for $\bar{r}_{11}, \bar{r}_{12}, \bar{r}_{22}$, where $\bar{r}$ is the position vector of any point of a surface and suffixes 1 and 2 denotes differentiation with regard to u and v respectively. 6

(D) If K, K_n denotes the curvature of oblique and normal sections through the same tangent line and θ be the angle between the sections then prove :

$$K_n = K \cos \theta.$$ 6

UNIT—IV

4\. (A) Prove that two geodesics at right angles have their torsions equal in magnitude but opposite in sign. 6

(B) Prove Bonet's theorem for a curve on a surface that $w' + \tau = \tau_g$, where w is the normal angle and τ_g is the torsion of the geodesic tangent. 6

OR

4\. (C) Obtain the differential equation of geodesic on a surface of revolution $Z = f\left(\sqrt{x^2 + y^2}\right)$ and deduce that on a right cylinder the geodesics are helices. 6

(D) Prove that the metric of a surface can always be reduced to $ds^2 = du^2 + G(u, v)\, dv^2$ with the auxiliary conditions :

(i) $\sqrt{G(o,v)} = 1, \quad \left[\frac{\partial}{\partial u}\sqrt{G(u,v)}\right]_{u=0} = 1;$

(ii) $\sqrt{G(o,v)} = 0, \quad \left[\frac{\partial}{\partial u}\sqrt{G(u,v)}\right]_{u=0} = 1.$ 6

Question—V

5. (A) Define principal normal and binormal to a curve $\bar{r} = \bar{r}(s)$. 1½

(B) Define Fundamental plane at a point P whose position vector is $\bar{r}$ on the space curve $\bar{R}$. 1½

(C) State fundamental theorem of space curves. 1½

(D) Define the envelope of the family if $(x, y, z, \theta) = 0$. 1½

(E) Define Gaussian curvature of a surface at any point P. 1½

(F) Define a third fundamental form. 1½

(G) A particle is constrained to move on a smooth surface under no force accept the normal reaction. Prove that its path is a geodesic. 1½

(H) Define geodesic polar coordinates for the geodesic metric $ds^2 = du^2 + G(u, v)\, dv^2$. 1½

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Bachelor of Science (B.Sc.) Semester–VI
(C.B.S.) Examination
MATHEMATICS
(M-12 : Special Theory of Relativity)
Optional Paper—2

Time—Three Hours] [Maximum Marks—60

N.B. :— (1) Solve all the **FIVE** questions.

(2) All questions carry equal marks.

(3) Question Nos. **1** to **4** have an alternative. Solve each question in full or its alternative in full.

UNIT—I

1. (A) Derive general and simple or special Galilean transformation by considering two inertial frames S and S'. Also obtain their inverse transformations. 6

(B) Show that :

$$(ds)^2 = (dx)^2 + (dy)^2 + (dz)^2 - (cdt)^2$$

is invariant under Lorentz transformations. 6

OR

1. (C) Explain Lorentz-Fitzgerald Contraction Idea. How was this idea used to account for the negative result of Michelson-Morley Experiment ? 6

(D) Show that the three dimensional volume element dx dy dz is not Lorentz invariant but the four dimensional volume elements dx dy dz dt is Lorentz invariant. 6

UNIT—II

2. (A) Obtain the transformation equations for components of particle velocities by using Lorentz transformations. Also write their inverse transformation. 6

(B) Explain the phenomenon of "Length Contraction" in special theory of relativity. 6

OR

2. (C) Let $\bar{u}$ and $\bar{u}'$ be the velocities of a particle in two inertial systems S and S' respectively, where S' is moving with velocity v relative to S along the XX'-axis. Show that :

$$\tan\theta' = \frac{\sin\theta\left(1-\frac{v^2}{c^2}\right)^{1/2}}{\cos\theta-\frac{v}{u}}$$

and $u'^2 = \dfrac{u^2\left\{1-2\frac{v}{u}\cos\theta+\left(\frac{v}{u}\right)^2-\left(\frac{v}{c}\right)^2\sin\theta\right\}}{\left(1-\frac{uv}{c^2}\cos\theta\right)^2}$

where θ and θ' are the angle between the X-axis and the vectors $\bar{u}$ and $\bar{u}'$ respectively. 6

(D) Prove that 'Simultaneity has only a relative and not an absolute meaning in special relativity. 6

UNIT—III

3. (A) Show that $g_{\mu\nu}$ is a covariant symmetric tensor of order 2. 6

(B) Find (a) g and (b) g^{ij} corresponding to the line element :

$$ds^2 = d\rho^2 + \rho^2 d\phi^2 + dz^2$$

in terms of cylindrical coordinates ρ, ϕ and z. 6

OR

3. (C) Define Four vector. Show that $A^1 = -A_1$, $A^2 = -A_2$, $A^3 = -A_3$, $A^4 = A_4$. Also show that the square of the length of a four vector is invariant under Lorentz transformation. 6

(D) Define Four tensor. Obtain the transformations of the components of a symmetrical four tensor T^{11} under the Lorentz transformation. 6

UNIT—IV

4. (A) Obtain the mass energy equivalence $E = mc^2$. 6

(B) Obtain the transformation equations for Momentum and Energy. 6

OR

4. (C) State the Maxwell's equations of electromagnetic theory in vaccum. Derive the wave equation for the propagation of the electric field strength $\overline{E}$ and the magnetic field strength $\overline{H}$ in free space with velocity of light. 6

(D) Prove that the energy-momentum tensor of electromagnetic field is symmetric and also it is trace free. 6

Question—V

5. (A) State the fundamental postulates of special relativity. 1½

(B) Show that the circle $x'^2 + y'^2 = a^2$ in the frame of reference S' is measured to be an ellipse in S if S' moves with uniform velocity relative to S. 1½

(C) Derive Einstein's velocity addition law. 1½

(D) Two particles move towards each other with speed 0.8.c. What is their relative speed ? 1½

(E) Prove that Kronecker delta (δ^i_j) is a mixed tensor of rank two. 1½

(F) If $A_\alpha = g_{\alpha\beta} A^\beta$, then show that $A^\beta = g^{\alpha\beta} A_\alpha$. 1½

(G) Define four velocity and four acceleration. 1½

(H) Show that the four-velocity of a particle is a unit time like vector. 1½